MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

DEM5038 – ENGINEERING MATHEMATICS 3

(Group E9)

12 OCTOBER 2017
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **11** pages (**4** pages with **5** questions and 7 pages for appendix).

2. Answer **ALL** questions. All necessary working steps must be shown.

3. Write all your answers in the answer booklet provided.

Please answer ALL questions and show the necessary working. Total mark is equal to 100.

Question 1

a) Find the general solution to the following non-homogenous differential equation. $y^{iv} + y''' - 3y'' - 5y' - 2y = -9e^{-x}$ [15 marks]

b) Find the solution to the following differential equation by using **Laplace transform** method.

$$y''+6y'+8y = 16, \quad y(0) = 0, \; y'(0) = 10$$ [12 marks]

[Total = 27 marks]

Question 2

a) The Figure 1 below is showing one period of a periodic function.

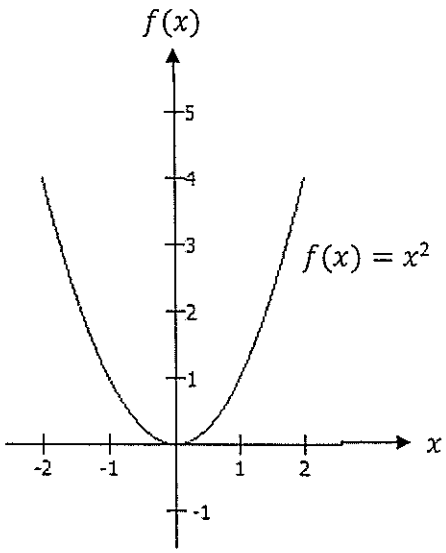


Figure 1

Find the Fourier series for the above function. [10 marks]

b) Find the Half-range **Sine** series for

$$f(x) = \pi x, \qquad 0 \le x \le \pi$$ [9 marks]

[Total = 19 marks]

Continued...

Question 3

a) Determine the Laplace transform of the following functions.

i. $e^{-3t}\cos 5t$ [2 marks]

ii. $5t^3e^t - 2t^4$ [2 marks]

iii. $\int_0^t 3\sinh u\,du$ [2 marks]

iv. $f(t) = \begin{cases} t-4, & 0 < t < 4 \\ 4, & t > 4 \end{cases}$ [4 marks]

b) Find the **inverse** Laplace transform for the following functions.

i. $\dfrac{20}{(s+4)^5}$ [2 marks]

ii. $\dfrac{2}{(s-3)^2+16}$ [2 marks]

iii. $\dfrac{8s^3+10s^2}{s^5}$ [2 marks]

iv. $\dfrac{s+4}{s^2+9}$ [3 marks]

[Total = 19 marks]

Continued...

Question 4

a) Table 1 gives the two-way classification of 60 students based on their status and their major.

Table 1

	Law	Engineering	IT	Business
Alpha	7	6	7	3
Beta	1	9	0	9
Gamma	3	2	5	8

One student is selected randomly from this group of 60 students. What is the probability that this student:

i. is a Beta student? [1 mark]

ii. is not an Engineering student? [2 marks]

iii. is a Beta student given that his/her major is Engineering? [2 marks]

iv. is majoring in Business or in Alpha year? [2 marks]

b) According to a production engineer, a trained worker should be able to assemble a certain type of chair in 15 minutes. A recent 150 randomly selected chairs showed that the average assembly time taken to assemble them is 15.4 minutes with standard deviation of 2.4 minutes. Can you conclude that the average assembly time to assemble a chair is different from 15 minutes using significance level of 0.05?

[8 marks]

[Total = 15 marks]

Continued...

Question 5

a) A study conducted shows that 36% of all busses arriving at Melaka Central are late. Using the binomial distribution function, find the probability that in a random sample of seven busses,

i. at least two will arrive late. [2 marks]

ii. from 1 to 4 busses will arrive late. [2 marks]

b) The distribution for the weight of fish in South China Sea can be regarded as normal with mean 3.7 kg and standard deviation 1.8 kg. Determine the probability that a fish caught in this sea has a weight,

i. between 3.5 kg and 5 kg. [2 marks]

ii. more than 4 kg. [2 marks]

c) A company analyzed the number of order and amount of overtime in hours for 8 weeks, with the following results tabulated in Table 2 as shown.

Table 2

Week	Number of orders	Hours of overtime
1	120	84
2	85	40
3	60	0
4	72	12
5	130	74
6	94	50
7	66	15
8	80	45

i. Determine the correlation coefficient between the number of orders and hours of overtime. [9 marks]

ii. Find the estimate regression line of y on x, $\hat{y} = a + bx$. [3 marks]

[Total = 20 marks]

End of Page.

Appendix

Quadratic formula $m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annihilator for special functions

Annihilator	
D^n	$1, x, x^2, x^3, x^4, \ldots\ldots, x^{n-1}$
$(D-a)^n$	$e^{ax}, xe^{ax}, x^2e^{ax}, x^3e^{ax}, x^4e^{ax}, \ldots\ldots, x^{n-1}e^{ax}$
$\left(D^2 - 2aD + (a^2+b^2)\right)^n$	$e^{ax}\cos bx, xe^{ax}\cos bx, x^2e^{ax}\cos bx, x^3e^{ax}\cos bx, \ldots\ldots, x^{n-1}e^{ax}\cos bx$ and $e^{ax}\sin bx, xe^{ax}\sin bx, x^2e^{ax}\sin bx, x^3e^{ax}\sin bx, \ldots\ldots, x^{n-1}e^{ax}\sin bx$

Fourier Series

$$F(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x\right)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi}{L}xdx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi}{L}xdx$$

Integration Formula

$$\int \sin axdx = -\frac{1}{a}\cos ax$$

$$\int \cos axdx = \frac{1}{a}\sin ax$$

$$\int x\sin axdx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos axdx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin axdx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos axdx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

Linear Regression and Correlation

$$\hat{y} = a + bx$$

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} \qquad SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} \qquad SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$b = \frac{SS_{xy}}{SS_{xx}} \qquad a = \bar{y} - b\bar{x} = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

Transformation of some functions

$f(t)$	$L\{f(t)\}$
1	$\frac{1}{s}$, s>0
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$ $n = 1,2,3,\ldots$
e^{at}	$\frac{1}{s-a}$, s>a
$t\,e^{at}$	$\frac{1}{(s-a)^2}$
y'	$sY(s)-y(0)$
y''	$s^2Y(s)-sy(0)-y'(0)$

$f(t)$	$L\{f(t)\}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
$(1)u(t-a)$	$\frac{e^{-sa}}{s}$
$(t-a)u(t-a)$	$\frac{e^{-sa}}{s^2}$

Second Shift Theorem

$$L\{f(t-a)u(t-a)\} = e^{-as}L\{f(t)\}$$
$$L\{f(t)u(t-a)\} = e^{-as}L\{f(t+a)\}$$

Binomial Probabilities $\binom{n}{x}p^x q^{n-x}$

Binomial Formulae when using Cambridge Statistical Table

Key Formulas (if $p \le 0.5$)

1. $P(X=r) = B(r) - B(r-1)$
2. $P(X \ge r) = 1 - B(r-1)$
3. $P(X > r) = 1 - B(r)$
4. $P(a \le X \le b) = B(b) - B(a-1)$
5. $P(X \le r) = B(r)$

Key Formulas (if $p > 0.5$)

1. $P(X=r) = P(Y=n-r) = B(n-r) - B(n-r-1)$
2. $P(X \ge r) = P(Y \le n-r) = B(n-r)$
3. $P(X \le r) = P(Y \ge n-r) = 1 - B(n-r-1)$
4. $P(a \le X \le b) = P(n-b \le Y \le n-a)$
   $= B(n-a) - B(n-b-1)$

Normal Distribution $Z = \frac{X-\mu}{\sigma}$

Hypothesis Testing $S_x = \frac{s}{\sqrt{n}}$ $Z = \frac{\bar{x}-\mu}{S_x}$

TABLE 1. THE BINOMIAL DISTRIBUTION FUNCTION

$n = 4$	$r = 0$	1	2	3
$p = 0.01$	0.9606	0.9994		
.02	.9224	.9977		
.03	.8853	.9948	0.9999	
.04	.8493	.9909	.9998	
0.05	0.8145	0.9860	0.9995	
.06	.7807	.9801	.9992	
.07	.7481	.9733	.9987	
.08	.7164	.9656	.9981	
.09	.6857	.9570	9973	0.9999
0.10	0.6561	0.9477	0.9963	0.9999
.11	.6274	.9376	.9951	.9999
.12	.5997	.9268	.9937	9998
.13	.5729	.9153	.9921	.9997
.14	.5470	.9032	.9902	.9996
0.15	0.5220	0.8905	0.9880	0.9995
.16	.4979	.8772	.9856	.9993
.17	.4746	.8634	.9829	.9992
.18	.4521	.8491	.9798	.9990
.19	.4305	.8344	.9765	.9987
0.20	0.4096	0.8192	0.9728	0.9984
.21	.3895	.8037	.9688	.9981
.22	.3702	.7878	.9644	.9977
.23	.3515	.7715	.9597	.9972
.24	.3336	.7550	.9547	.9967
0.25	0.3164	0.7383	0.9492	0.9961
.26	.2999	.7213	.9434	.9954
.27	.2840	.7041	.9372	.9947
.28	.2687	6868	.9306	9939
.29	.2541	.6693	.9237	.9929
0.30	0.2401	0.6517	0.9163	0.9919
.31	.2267	6340	.9085	.9908
.32	.2138	.6163	.9004	.9895
.33	.2015	.5985	.8918	.9881
.34	.1897	.5807	.8829	.9866
0.35	0.1785	0.5630	0.8735	0.9850
.36	.1678	.5453	.8638	.9832
.37	.1575	.5276	.8536	.9813
.38	.1478	.5100	.8431	.9791
.39	.1385	.4925	.8321	.9769
0.40	0.1296	0.4752	0.8208	0.9744
.41	.1212	.4580	.8091	.9717
.42	.1132	.4410	.7970	.9689
.43	.1056	.4241	.7845	.9658
.44	.0983	.4074	.7717	.9625
0.45	0.0915	0.3910	0.7585	0.9590
.46	.0850	.3748	.7450	.9552
.47	.0789	.3588	.7311	.9512
.48	.0731	.3431	.7169	.9469
.49	.0677	.3276	.7023	.9424
0.50	0.0625	0.3125	0.6875	0.9375

$n = 5$	$r = 0$	1	2	3	4
$p = 0.01$	0.9510	0.9990			
.02	.9039	.9962	0.9999		
.03	.8587	.9915	.9997		
.04	.8154	.9852	.9994		
0.05	0.7738	0.9774	0.9988		
.06	.7339	.9681	.9980	0.9999	
.07	.6957	.9575	.9969	.9999	
.08	.6591	.9456	.9955	.9998	
.09	.6240	.9326	.9937	.9997	
0.10	0.5905	0.9185	0.9914	0.9995	
.11	.5584	.9035	.9888	.9993	
.12	.5277	.8875	.9857	.9991	
.13	.4984	.8708	.9821	.9987	
.14	.4704	.8533	.9780	.9983	0.9999
0.15	0.4437	0.8352	0.9734	0.9978	0.9999
.16	.4182	.8165	.9682	.9971	.9999
.17	.3939	.7973	.9625	.9964	.9999
.18	.3707	.7776	.9563	.9955	.9998
.19	.3487	.7576	.9495	.9945	.9998
0.20	0.3277	0.7373	0.9421	0.9933	0.9997
.21	.3077	.7167	.9341	.9919	.9996
.22	.2887	.6959	.9256	.9903	.9995
.23	.2707	.6749	.9164	.9886	.9994
.24	.2536	.6539	.9067	.9866	.9992
0.25	0.2373	0.6328	0.8965	0.9844	0.9990
.26	.2219	.6117	.8857	.9819	.9988
.27	.2073	.5907	.8743	.9792	.9986
.28	.1935	.5697	.8624	.9762	.9983
.29	.1804	.5489	.8499	.9728	.9979
0.30	0.1681	0.5282	0.8369	0.9692	0.9976
.31	.1564	.5077	.8234	.9653	.9971
.32	.1454	.4875	.8095	.9610	.9966
.33	.1350	.4675	.7950	.9564	.9961
.34	.1252	.4478	.7801	.9514	.9955
0.35	0.1160	0.4284	0.7648	0.9460	0.9947
.36	.1074	.4094	.7491	.9402	.9940
.37	.0992	.3907	.7330	.9340	.9931
.38	.0916	.3724	.7165	.9274	.9921
.39	.0845	.3545	.6997	.9204	.9910
0.40	0.0778	0.3370	0.6826	0.9130	0.9898
.41	.0715	.3199	.6651	.9051	.9884
.42	.0656	.3033	.6475	.8967	.9869
.43	.0602	.2871	.6295	.8879	.9853
.44	.0551	.2714	.6114	.8786	.9835
0.45	0.0503	0.2562	0.5931	0.8688	0.9815
.46	.0459	.2415	.5747	.8585	.9794
.47	.0418	.2272	.5561	.8478	.9771
.48	.0380	.2135	.5375	.8365	.9745
.49	.0345	.2002	.5187	.8248	.9718
0.50	0.0313	0.1875	0.5000	0.8125	0.9688

See page 4 for explanation of the use of this table.

TABLE 1. THE BINOMIAL DISTRIBUTION FUNCTION

$n=6$	$r=0$	1	2	3	4	5
$p=0.01$	0.9415	0.9985				
.02	.8858	.9943	0.9998			
.03	.8330	.9875	9995			
.04	.7828	.9784	.9988			
0.05	0.7351	0.9672	0.9978	0.9999		
.06	.6899	.9541	.9962	.9998		
.07	.6470	.9392	.9942	.9997		
.08	.6064	.9227	.9915	.9995		
.09	.5679	.9048	9882	.9992		
0.10	0.5314	0.8857	0.9842	0.9987	0.9999	
.11	.4970	.8655	.9794	.9982	.9999	
.12	.4644	.8444	.9739	.9975	9999	
.13	.4336	.8224	.9676	.9966	.9998	
.14	.4046	.7997	9605	.9955	.9997	
0.15	0.3771	0.7765	0.9527	0.9941	0.9996	
.16	.3513	.7528	.9440	.9925	.9995	
.17	.3269	.7287	9345	.9906	.9993	
.18	.3040	.7044	.9241	.9884	.9990	
.19	.2824	.6799	9130	.9859	.9987	
0.20	0.2621	0.6554	0.9011	0.9830	0.9984	0.9999
.21	.2431	.6308	.8885	.9798	.9980	.9999
.22	.2252	.6063	.8750	.9761	9975	9999
.23	.2084	.5820	.8609	.9720	.9969	.9999
.24	.1927	.5578	.8461	.9674	.9962	.9998
0.25	0.1780	0.5339	0.8306	0.9624	0.9954	0.9998
.26	.1642	.5104	.8144	.9569	.9944	.9997
.27	.1513	.4872	.7977	.9508	.9933	.9996
.28	.1393	.4644	.7804	.9443	.9921	.9995
.29	.1281	.4420	.7626	.9372	.9907	.9994
0.30	0.1176	0.4202	0.7443	0.9295	0.9891	0.9993
.31	.1079	.3988	.7256	.9213	.9873	.9991
.32	.0989	.3780	.7064	.9125	.9852	.9989
.33	.0905	.3578	.6870	.9031	.9830	.9987
.34	.0827	.3381	.6672	.8931	.9805	.9985
0.35	0.0754	0.3191	0.6471	0.8826	0.9777	0.9982
.36	.0687	.3006	.6268	.8714	.9746	.9978
.37	.0625	.2828	.6063	.8596	.9712	.9974
.38	.0568	.2657	.5857	.8473	.9675	.9970
.39	.0515	.2492	.5650	.8343	.9635	.9965
0.40	0.0467	0.2333	0.5443	0.8208	0.9590	0.9959
.41	.0422	.2181	.5236	.8067	.9542	.9952
.42	.0381	.2035	.5029	.7920	.9490	.9945
.43	.0343	.1895	.4823	.7768	.9434	.9937
.44	.0308	.1762	.4618	.7610	.9373	.9927
0.45	0.0277	0.1636	0.4415	0.7447	0.9308	0.9917
.46	.0248	.1515	.4214	.7279	.9238	.9905
.47	.0222	.1401	.4015	.7107	.9163	.9892
.48	.0198	.1293	.3820	.6930	.9083	.9878
.49	.0176	.1190	.3627	.0748	.8997	9862
0.50	0.0156	0.1094	0.3438	0.6562	0.8906	0.9844

$n=7$	$r=0$	1	2	3
$p=0.01$	0.9321	0.9980		
.02	.8681	.9921	0.9997	
.03	.8080	.9829	.9991	
.04	.7514	.9706	.9980	0.9999
0.05	0.6983	0.9556	0.9962	0.9998
.06	.6485	.9382	.9937	.9996
.07	.6017	.9187	.9903	.9993
.08	.5578	.8974	.9860	.9988
.09	.5168	.8745	.9807	.9982
0.10	0.4783	0.8503	0.9743	0.9973
.11	.4423	.8250	.9669	.9961
.12	.4087	.7988	.9584	.9946
.13	.3773	.7719	.9487	.9928
.14	.3479	.7444	.9380	.9906
0.15	0.3206	0.7166	0.9262	0.9879
.16	.2951	.6885	.9134	.9847
.17	.2714	.6604	.8995	.9811
.18	.2493	.6323	.8846	.9769
.19	.2288	.6044	.8687	.9721
0.20	0.2097	0.5767	0.8520	0.9667
.21	.1920	.5494	.8343	.9606
.22	.1757	.5225	8159	.9539
.23	.1605	.4960	.7967	.9464
.24	.1465	.4702	.7769	.9383
0.25	0.1335	0.4449	0.7564	0.9294
.26	.1215	.4204	.7354	.9198
.27	.1105	.3965	.7139	.9095
.28	.1003	.3734	.6919	.8984
.29	.0910	.3510	.6696	.8866
0.30	0.0824	0.3294	0.6471	0.8740
.31	.0745	.3086	.6243	.8606
.32	.0672	.2887	.6013	.8466
.33	.0606	.2696	.5783	.8318
.34	.0546	.2513	.5553	.8163
0.35	0.0490	0.2338	0.5323	0.8002
.36	.0440	.2172	.5004	.7833
.37	.0394	.2013	.4866	.7659
.38	.0352	.1863	.4641	.7479
.39	.0314	.1721	.4419	.7293
0.40	0.0280	0.1586	0.4199	0.7102
.41	.0249	.1459	.3983	.6906
.42	.0221	.1340	.3771	.6706
.43	.0195	.1228	.3564	.6502
.44	.0173	.1123	.3362	.6294
0.45	0.0152	0.1024	0.3164	0.6083
.46	.0134	.0932	.2973	.5869
.47	.0117	.0847	.2787	.5654
.48	.0103	.0767	.2607	.5437
.49	.0090	.0693	.2433	.5219
0.50	0.0078	0.0625	0.2266	0.5000

See page 4 for explanation of the use of this table.

TABLE I. THE BINOMIAL DISTRIBUTION FUNCTION

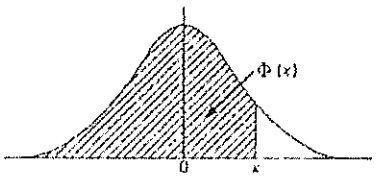
$n = 7$	$r = 4$	5	6
$p = 0.01$			
.02			
.03			
.04			
0.05			
.06			
.07			
.08	0.9999		
.09	.9999		
0.10	0.9998		
.11	.9997		
.12	.9996		
.13	.9994		
.14	.9991		
0.15	0.9988	0.9999	
.16	.9983	.9999	
.17	.9978	.9999	
.18	.9971	.9998	
.19	.9963	.9997	
0.20	0.9953	0.9996	
.21	.9942	.9995	
.22	.9928	.9994	
.23	.9912	.9992	
.24	.9893	.9989	
0.25	0.9871	0.9987	0.9999
.26	.9847	.9983	.9999
.27	.9819	.9979	.9999
.28	.9787	.9974	.9999
.29	.9752	.9969	.9998
0.30	0.9712	0.9962	0.9998
.31	.9668	.9954	.9997
.32	.9620	.9945	.9997
.33	.9566	.9935	.9996
.34	.9508	.9923	.9995
0.35	0.9444	0.9910	0.9994
.36	.9375	.9895	.9992
.37	.9299	.9877	.9991
.38	.9218	.9858	.9989
.39	.9131	.9836	.9986
0.40	0.9037	0.9812	0.9984
.41	.8937	.9784	.9981
.42	.8831	.9754	.9977
.43	.8718	.9721	.9973
.44	.8598	.9684	.9968
0.45	0.8471	0.9643	0.9963
.46	.8337	.9598	.9956
.47	.8197	.9549	.9949
.48	.8049	.9496	.9941
.49	.7895	.9438	.9932
0.50	0.7734	0.9375	0.9922

$n = 8$	$r = 0$	1	2	3	4	5	6
$p = 0.01$	0.9227	0.9973	0.9999				
.02	.8508	.9897	.9996				
.03	.7837	.9777	.9987	0.9999			
.04	.7214	.9619	.9969	.9998			
0.05	0.6634	0.9428	0.9942	0.9996			
.06	.6096	.9208	.9904	.9993			
.07	.5596	.8965	.9853	.9987	0.9999		
.08	.5132	.8702	.9789	.9978	.9999		
.09	.4703	.8423	.9711	.9966	.9997		
0.10	0.4305	0.8131	0.9619	0.9950	0.9996		
.11	.3937	.7829	.9513	.9929	.9993		
.12	.3596	.7520	.9392	.9903	.9990	0.9999	
.13	.3282	.7206	.9257	.9871	.9985	.9999	
.14	.2992	.6889	.9109	.9832	.9979	.9998	
0.15	0.2725	0.6572	0.8948	0.9786	0.9971	0.9998	
.16	.2479	.6256	.8774	.9733	.9962	.9997	
.17	.2252	.5943	.8588	.9672	.9950	.9995	
.18	.2044	.5634	.8392	.9603	.9935	.9993	
.19	.1853	.5330	.8185	.9524	.9917	.9991	0.9999
0.20	0.1678	0.5033	0.7969	0.9437	0.9896	0.9988	0.9999
.21	.1517	.4743	.7745	.9341	.9871	.9984	.9999
.22	.1370	.4462	.7514	.9235	.9842	.9979	.9998
.23	.1236	.4189	.7276	.9120	.9809	.9973	.9998
.24	.1113	.3925	.7033	.8996	.9770	.9966	.9997
0.25	0.1001	0.3671	0.6785	0.8862	0.9727	0.9958	0.9996
.26	.0899	.3427	.6535	.8719	.9678	.9948	.9995
.27	.0806	.3193	.6282	.8567	.9623	.9936	.9994
.28	.0722	.2969	.6027	.8406	.9562	.9922	.9992
.29	.0646	.2756	.5772	.8237	.9495	.9906	.9990
0.30	0.0576	0.2553	0.5518	0.8059	0.9420	0.9887	0.9987
.31	.0514	.2360	.5264	.7874	.9339	.9866	.9984
.32	.0457	.2178	.5013	.7681	.9250	.9841	.9980
.33	.0406	.2006	.4764	.7481	.9154	.9813	.9976
.34	.0360	.1844	.4519	.7276	.9051	.9782	.9970
0.35	0.0319	0.1691	0.4278	0.7064	0.8939	0.9747	0.9964
.36	.0281	.1548	.4042	.6847	.8820	.9707	.9957
.37	.0248	.1414	.3811	.6626	.8693	.9664	.9949
.38	.0218	.1289	.3585	.6401	.8557	.9615	.9939
.39	.0192	.1172	.3366	.6172	.8414	.9561	.9928
0.40	0.0168	0.1064	0.3154	0.5941	0.8263	0.9502	0.9915
.41	.0147	.0963	.2948	.5708	.8105	.9437	.9900
.42	.0128	.0870	.2750	.5473	.7938	.9366	.9883
.43	.0111	.0784	.2560	.5238	.7765	.9289	.9864
.44	.0097	.0705	.2376	.5004	.7584	.9206	.9843
0.45	0.0084	0.0632	0.2201	0.4770	0.7396	0.9115	0.9819
.46	.0072	.0565	.2034	.4537	.7202	.9018	.9792
.47	.0062	.0504	.1875	.4306	.7001	.8914	.9761
.48	.0053	.0448	.1724	.4078	.6795	.8802	.9728
.49	.0046	.0398	.1581	.3854	.6584	.8682	.9690
0.50	0.0039	0.0352	0.1445	0.3633	0.6367	0.8555	0.9648

See page 4 for explanation of the use of this table.

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990
3·105	0·9991	3·320	0·9995	3·759	0·99991
3·138	0·9992	3·389	0·9996	3·791	0·99992
3·174	0·9993	3·480	0·9997	3·826	0·99993
3·215	0·9994	3·615	0·9998	3·867	0·99994
			0·9999		0·99995

3·916	0·99995
3·976	0·99996
4·055	0·99997
4·173	0·99998
4·417	0·99999
	1·00000

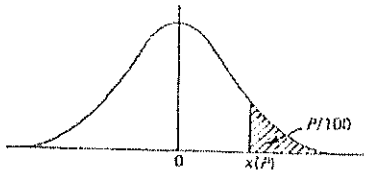
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteq \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172